

# Composite Materials: Analysis and Design

## Homework no.4

Due Date: 3th May

### Problem 1

Fig. 1 illustrates a rectangular array of elliptical-shaped fibers. Determine: a) the relationship between the fiber volume fraction and the specified geometrical parameters, b) the maximum possible fiber volume fraction for this packing geometry.

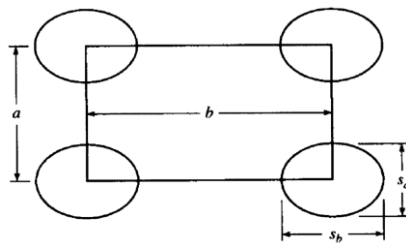


Fig. 1

### Problem 2

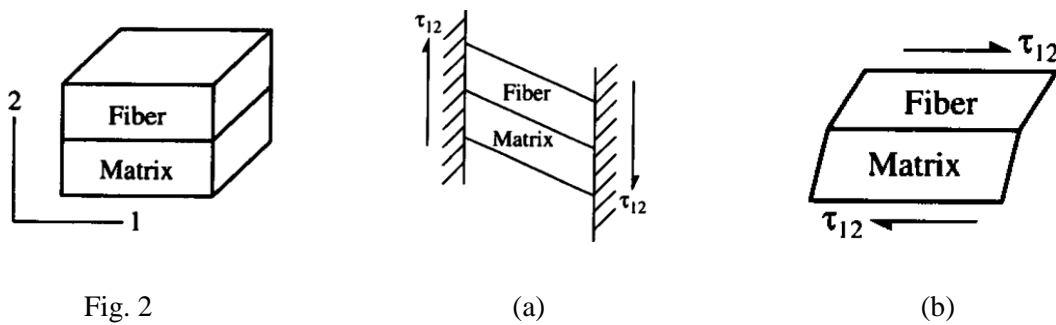
Homogeneous engineering materials are infrequent. Consider a composite which consists of a small spherical particle of one material that is embedded in a large matrix of another material. The particle has a radius of  $a$ . The composite is solidified at an elevated temperature  $T_h$ , at which the composite is in a stress-free condition. The particle has a larger thermal expansion coefficient than the matrix,  $\alpha_p > \alpha_m$ . Upon cooling process, the composite develops a residual stress field because of the thermal expansion mismatch.

- At room temperature  $T_r$ , is the particle under tension or compression?
- Find out the complete stress field in the particle and the matrix at room temperature. Assume that both the materials are isotropic. To simplify algebra, assume the same Young's modulus  $E$  and the same Poisson's ratio  $\nu$  for both the materials.

### **Problem 3**

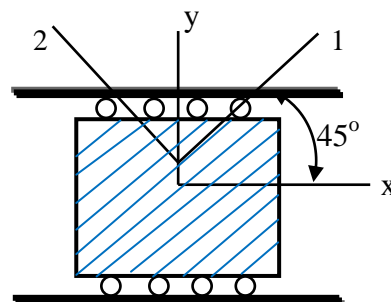
Fig. 2 presents a unidirectional composite, in which the fiber and matrix materials are assumed to be isotropic and perfectly bonded together. By using a mechanics of materials approach, derive the micromechanics equations for the effective in-plane shear modulus,  $G_{12}$ , for these cases:

- (a) The ends of composite are perfectly bonded to supports which are restrained against rotation, then subjected to the uniform in-plane shear stress,  $\tau_{12}$  (Fig. 2-a).
- (b) Both the top and bottom surfaces of the composite are subjected to the uniform in-plane shear stress,  $\tau_{12}$ , and the ends of the composite are free to rotate, as depicted in Fig. 2-b.



### **Problem 4**

As shown in Fig. 4, a unidirectional  $45^\circ$  off-axis E-glass/epoxy composite lamina is supported on frictionless rollers between two rigid walls. The lamina is fixed against displacements in the  $y$  direction and is free to move in the  $x$  direction. The lamina is heated from  $20^\circ$  to  $120^\circ$ . Determine all of the lamina strains associated with  $x$ ,  $y$  axes.



**Fig. 3**